International Journal of Theoretical Physics, Vol. 34, No. 10, 1995

Constant-Cutoff Approach to $SU(3)$-Symmetry Breaking for Strange Dibaryon States

Nils Dalarsson[1]

Received February 3, 1995

We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then show that the approach to $SU(3)$-symmetry breaking for strange dibaryon states proposed by Kopeliovich *et al.* can be simplified by omitting the Skyrme stabilizing term and using the constant-cutoff stabilization method. We derive the results for spectra of some strange and nonstrange dibaryon states and obtain the numerical results for the absolute masses of these states, in reasonable agreement with the values obtained, using the complete Skyrme model, by Kopeliovich *et al.*

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \text{Tr}\, \partial_\mu U\, \partial^\mu U^+ \tag{1.1}$$

where

$$U = \frac{2}{F_\pi} (\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

[1] Royal Institute of Technology, Stockholm, Sweden.

0020-7748/95/1000-2129$07.50/0

The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1)

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2} \mathrm{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors (Adkins *et al.*, 1983; Witten, 1979, 1983a,b; for other references see Holzwarth and Schwesinger, 1986, and Nyman and Riska, 1990) that, after the collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (MW) (1989) indicated therefore the possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. MW showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4} F_\pi^2 \frac{1}{\chi''(0)} a + \frac{1}{2} \frac{[\chi''(0)]^3}{(\pi/4)F_\pi^2 b} J(J + 1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2\left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4} \mathcal{F}\right)\right] ds \tag{1.6}$$

$$b = \int_0^\infty ds \frac{64}{3} s^2 \sin^2\left(\frac{1}{4} \mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \frac{1}{4} \mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of a stable classical soliton solution to the nonlinear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by Mignaco and Wulck (1989) is not correct, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. If the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + (cy)^{1/2} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha\right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$, then, we want to know which of these two solutions is approached by $F(y)$ when $y \to 0$ $(r \to \infty)$. In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, (1.13) reduces to

$$\int_0^\infty 2y[F'(y)]^2\, dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ $(r \to 0)$ is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\, dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-runoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991) and Jain *et al.* (1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to $SU(3)$-symmetry breaking in strange dibaryon states and derive the results for spectra of some strange and non-strange dibaryon states and obtain the numerical results for the masses of these states.

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary conditions $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991; see also Jain *et al.*, 1989), when a cutoff ϵ is introduced, then the boundary conditions

$F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. In Balakrishna *et al.* (1991) an interesting analogy with the damped pendulum is discussed, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced, following Balakrishna *et al.* (1991), as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U\, \partial^\mu U^+)\, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$\begin{aligned} U(\mathbf{r}, t) &= A(t) U_0(\mathbf{r}, t) A^+(t) \\ U^+(\mathbf{r}, t) &= A(t) U_0^+(\mathbf{r}, t) A^+(t) \end{aligned} \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{ i\boldsymbol{\tau} \cdot \mathbf{r}_0 F[r; \epsilon(t)] \} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3\mathbf{r} = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3\mathbf{r} = bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[\dot{x}(t)]^2 \qquad (2.7)$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2\, dy$$

$$c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2\, dy \qquad (2.8)$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2 \left(\frac{dF}{dy}\right)^2 + 2\sin^2 F \right] dy \qquad (2.9)$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu \qquad (2.10)$$

where $\mathrm{Tr}(\partial_0 A\ \partial_0 A^+) = 2\dot{\alpha}_\nu \dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in (Bhaduri, 1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu} \dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2 \dot{\alpha}_\nu \dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \qquad (2.11)$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3} \frac{ab}{J(J+1)}\right]^{1/4} \qquad (2.12)$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3} \left[\frac{3}{2} \frac{a^3}{b} J(J+1)\right]^{1/4} \qquad (2.13)$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to (\pi/4)F_\pi^2 a$, $b \to (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2} F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78\ \text{GeV}^2$, $b = 0.91\ \text{GeV}^2$, $c = 1.46\ \text{GeV}^2$ for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion-decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507\ \text{GeV}^2$ and $b = 0.592\ \text{GeV}^2$, giving the exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22\ \text{fm} & \text{for} \quad F_\pi = 186\ \text{MeV} \\ 0.27\ \text{fm} & \text{for} \quad F_\pi = 150\ \text{MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. THE *SU*(3)-EXTENDED SIMPLIFIED SKYRME MODEL

3.1. The Effective Interaction

The Lagrangian density for a dibaryon system with pseudoscalar mesons is, with Skyrme stabilizing term omitted, given by (Callan and Klebanov, 1985; Callan *et al.*, 1988; Kunz and Mulders, 1988; Kopeliovich *et al.*, 1992)

$$\begin{aligned} \mathcal{L} &= \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U\, \partial^\mu U^+ + \frac{F_\pi^2}{16} m_\pi^2 \operatorname{Tr}(U + U^+ - 2) \\ &\quad - \frac{1}{48} (F_K^2 - F_\pi^2) \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U\, \partial_\mu U\, \partial^\mu U^+ + \partial_\mu U\, \partial^\mu U^+ U^+) \\ &\quad + \frac{1}{24} (F_K^2 m_K^2 - F_\pi^2 m_\pi^2) \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U + U^+ - 2) \end{aligned} \tag{3.1}$$

where m_π and m_K are pion and kaon masses, respectively, and F_K is the kaon week-decay constant with the empirical value $F_K = 226$ MeV. The first term in (3.1) is the usual σ-model Lagrangian, while the remaining three terms are all chiral- and flavor-symmetry-breaking terms, present in the mesionic sector of the model, which will be used in this form even for the multibaryon ($n > 1$) states. All flavor-symmetry-breaking terms in the effective Lagrangian (3.1) also break the chiral symmetry just as quark-mass terms do in the underlying QCD Lagrangian. In addition to the action, obtained using the Lagrangian (3.1), the Wess–Zumino action in the form

$$S = -\frac{iN_c}{240\pi^2}\int d^5x\, e^{\mu\nu\alpha\beta\gamma} \times \mathrm{Tr}[U^+\partial_\mu U\, U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U\, U^+\partial_\gamma U] \tag{3.2}$$

must be included into the total action of a dibaryon system, where N_c is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model, important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2.

In the single-baryon ($n = 1$) sector the lowest energy states have the hedgehog structure within $SU(2)$ given by (1.4). The lowest dibaryon states (Weigel *et al.*, 1985; Kopeliovich *et al.*, 1992) are characterized by an axially symmetric form of U leading to a torus-shaped baryon-number density, i.e., we have the $SU(2)$ ansatz

$$U_T(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\boldsymbol{\eta}\chi(r, z)]$$

$$\boldsymbol{\eta} = \begin{bmatrix} \sin\alpha(r, z) & \cos n\phi \\ \sin\alpha(r, z) & \sin n\phi \\ \multicolumn{2}{c}{\cos\alpha(r, z)} \end{bmatrix} \tag{3.3}$$

where $\chi = \chi(r, z)$ and $\alpha = \alpha(r, z)$ are the functions for the polar and chiral angles depending on two variables r and z, and ϕ is the azimuthal angle. The total soliton number of the torus-like state $U_T(\mathbf{r})$ given by (3.3) is $B = n$ if $\chi(\epsilon, 0) = \pi$. In the present paper the case $B = 2$ will be of primary interest. The extension to $SU(3)$ is done using the rotational ansatz

$$U(\mathbf{r}, t) = A(t)\begin{bmatrix} U_T[R_{ij}(t)r^j] & 0 \\ 0 & 1 \end{bmatrix}A^+(t) \tag{3.4}$$

with time-dependent $SU(3)$ rotation matrices

$$A(t) = I(\alpha, \beta, \gamma)\exp(-i\nu\lambda_4)\, I(\alpha', \beta', \gamma')\exp[-i(\rho/\sqrt{3})\lambda_8] \tag{3.5}$$

The matrices I in (3.5) denote rotations in the isospin subgroup of $SU(3)$ and

$R_{ij}(t)$ in (3.4) is a spatial rotation matrix. Flavor rotations increasing ν in (3.5) lead to an increasing absolute value of strangeness in a dibaryon state. The static mass of the torus is then obtained as a functional

$$M(\nu) = \frac{1}{8}\int_{r\geq\epsilon} d^3\mathbf{r}\left\{[F_\pi^2 + (F_K^2 - F_\pi^2)\sin^2\nu\cos\chi]\right.$$
$$\times\left[\left(\frac{\partial\chi}{\partial r}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial r}\right)^2 + \frac{n^2}{r^2}\sin^2\chi\sin^2\alpha\right.$$
$$\left. + \left(\frac{\partial\chi}{\partial z}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial z}\right)^2\right]$$
$$\left. + 2[F_\pi^2 m_\pi^2 + (F_K^2 m_K^2 - F_\pi^2 m_\pi^2)\sin^2\nu](1-\cos\chi)\right\} \tag{3.6}$$

The functional (3.6) is independent of the Euler angles except for a ν dependence originating from the flavor-symmetry-breaking terms of the effective action, and ϵ is the constant cutoff at the lower limit of the space interval $r \in [\epsilon, \infty]$.

3.2. The Rotational Energies of the Strange Dibaryons

For the time-dependent rotation matrices $A(t)$ and the spatial-rotation matrices $R(t)$ introduced in (3.4) we have the corresponding angular velocities θ and ω, respectively, defined by

$$A^{-1}\partial_0 A = \frac{1}{2}i\sum_{a=1}^{8}\lambda_a\theta^a, \qquad (R^{-1}\partial_0 R)_{mn} = \sum_{p=1}^{3}\epsilon_{mnp}\omega^p \tag{3.7}$$

where ϵ_{mnp} (m, n, p = 1, 2, 3) is the totally antisymmetric tensor with ϵ_{123} = 1. The Lagrangian of a rotated torus then becomes

$$L = -E(\nu) + \frac{1}{2}\Omega_N(\nu)\sum_{a=1}^{2}\theta_a\theta^a + \frac{1}{2}\Omega_j(\nu)\sum_{p=1}^{2}\omega_p\omega^p$$
$$+ \frac{1}{2}\Omega_3(\nu)(\theta_3 + n\omega_3)^2$$
$$+ \frac{1}{2}\Omega_s(\nu)\sum_{a=4}^{7}\theta_a\theta^a - \frac{N_c}{2\sqrt{3}}\theta_8^2 \tag{3.8}$$

As argued in Kopeliovich *et al.* (1992), for slowly rotating solitons the system has sufficient time to adjust its shape for every angle ν according to the forces exerted by the flavor-symmetry breaking. Such an assumption is certainly satisfied for N_c large and, as shown in Kopeliovich *et al.* (1992), it is also fairly well satisfied for $N_c = 3$. The moments of inertia in (3.8) are given by

$$\Omega_N(\nu) = \frac{1}{8}\int_{r\geq\epsilon} d^3\mathbf{r}\,[F_\pi^2 + \cos\chi\,\sin^2\nu\,(F_K^2 - F_\pi^2)] \times (1 + \cos^2\alpha)\sin^2\chi \tag{3.9}$$

$$\Omega_s(\nu) = \frac{1}{8}\int_{r\geq\epsilon} d^3\mathbf{r}\left[F_K^2 - (F_K^2 - F_\pi^2)\left(1 - \frac{1}{2}\cos\chi\right)\sin^2\nu\right] \times (1 - \cos\chi) \tag{3.10}$$

$$\Omega_3(\nu) = \frac{1}{4}\int_{r\geq\epsilon} d^3\mathbf{r}\,[F_\pi^2 + (F_K^2 - F_\pi^2)\cos\chi\,\sin^2\nu]\sin^2\chi\,\sin^2\alpha \tag{3.11}$$

$$\begin{aligned}\Omega_J(\nu) = \frac{1}{8}\int_{r\geq\epsilon} & d^3\mathbf{r}\,[F_\pi^2 + (F_K^2 - F_\pi^2)\cos\chi\,\sin^2\nu] \\ & \times \left\{z^2\left[\left(\frac{\partial\chi}{\partial r}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial r}\right)^2\right]\right. \\ & + n^2\frac{z^2}{r^2}\sin^2\chi\,\sin^2\alpha \\ & - 2rz\left(\frac{\partial\chi}{\partial r}\frac{\partial\chi}{\partial z} + \sin^2\chi\,\frac{\partial\chi}{\partial r}\frac{\partial\chi}{\partial z}\right) \\ & \left. + r^2\left[\left(\frac{\partial\chi}{\partial z}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial z}\right)^2\right]\right\}\end{aligned} \tag{3.12}$$

From equations (3.9), (3.11), and (3.12) we see that Ω_N, Ω_3, and Ω_J are easily obtained from the flavor-symmetric results, obtained in Braaten and Carlson (1988), by the substitution $F_\pi^2 \to F_\pi^2 + (F_K^2 - F_\pi^2)\cos\chi\,\sin^2\nu$, leading to a small increase of inertia.

After the collective-coordinate quantization the total Hamiltonian of the system becomes (Kopeliovich *et al.*, 1992)

$$\begin{aligned}H = M(\nu) + \frac{1}{2}\left[\frac{1}{2\Omega_s(\nu)}, C_2[SU(3)]\right] + \frac{1}{2\Omega_J(\nu)}J(J+1) \\ + \left[\frac{1}{2\Omega_N(\nu)} - \frac{1}{2\Omega_s(\nu)}\right]N(N+1)\end{aligned}$$

$$+ \left[\frac{1}{2\Omega_3(\nu)} - \frac{1}{2\Omega_N(\nu)} - \frac{n^2}{2\Omega_J(\nu)}\right]L^2 - \frac{3}{8\Omega_s(\nu)}B^2 \tag{3.13}$$

where the eigenvalues of diagonal operators are inserted. These eigenvalues also label the eigenfunctions of the Hamiltonian (3.13) given by (Kopeliovich *et al.*, 1990, 1992)

$$\Psi^{NL}_{YII_3,JJ_3}(A) = \sum_{M_L M_R} D^{(I)*}_{I_3 M_L}(\alpha, \beta, \gamma) f_{(M_L^{IY})(M_R^{N2})}(\nu) \times D^{(N)*}_{M_R L}(\alpha', \beta', \gamma') e^{2i\rho} D^{(J)*}_{J_3-nL}(\alpha', \beta', \gamma') \tag{3.14}$$

As argued in Kopeliovich *et al.* (1992) for the flavor-symmetric case, they are classified apart from their spin J, isospin I, and hypercharge Y by the definite $SU(3)$ representations (p, q) as

$$f_{(M_L^{IY})(M_R^{N2})}(\nu) = d^{(p,q)*}_{(M_L^{IY})(M_R^{N2})}(\nu) \tag{3.15}$$

leading to the $SU(3)$ rotation matrices given in Holland (1969). When the flavor symmetry is broken the angular eigenstates correspond to a mixture of different $SU(3)$ representations (p, q) leaving all other quantum numbers labeling Ψ in (3.14) unchanged. Among these there are also $N = \frac{1}{2}p_0$, where (p_0, q_0) is the representation of minimal triality $\frac{1}{3}(p + 2q)$ present in the mixture, and L, which determines the parity $P = (-)^L$ of the state.

3.3. The Spectrum of the Strange Dibaryons

Introducing now the dimensionless variables $\rho = r/\epsilon$, $\zeta = z/\epsilon$, where ϵ is the constant cutoff, into the integrals (3.6) and (3.9)–(3.12), we find that the Hamiltonian (3.13) becomes

$$H = a_1(\nu)\epsilon + a_2(\nu)\epsilon^3 + \beta(\nu)\epsilon^{-3} \tag{3.16}$$

where $a_1(\nu)$ and $a_2(\nu)$ are integrals given by

$$a_1(\nu) = \frac{1}{8}\int_{\rho\geq 1} d^3\boldsymbol{\rho}\, [F_\pi^2 + (F_K^2 - F_\pi^2)\sin^2\nu \cos\chi] \times \left[\left(\frac{\partial\chi}{\partial\rho}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial\rho}\right)^2 + \frac{n^2}{\rho^2}\sin^2\chi\,\sin^2\alpha + \left(\frac{\partial\chi}{\partial\zeta}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial\zeta}\right)^2\right] \tag{3.17}$$

$$a_2(\nu) = \frac{1}{4}\int_{\rho\geq 1} d^3\boldsymbol{\rho}\, [F_\pi^2 m_\pi^2 + (F_K^2 m_K^2 - F_\pi^2 m_\pi^2)\sin^2\nu](1 - \cos\chi) \tag{3.18}$$

and $\beta(\nu)$ is given by

$$\beta(\nu) = \frac{1}{2}\left[\frac{1}{2b_s(\nu)}, C_2[SU(3)]\right] + \frac{1}{2b_J(\nu)} J(J+1) + \left[\frac{1}{2b_N(\nu)} - \frac{1}{2b_s(\nu)}\right] N(N+1) + \left[\frac{1}{2b_3(\nu)} - \frac{1}{2b_N(\nu)} - \frac{n^2}{2b_J(\nu)}\right] L^2 - \frac{3}{8b_s(\nu)} B^2 \tag{3.19}$$

In (3.19), b_N, b_s, b_3, and b_J are integrals given by

$$b_N(\nu) = \frac{1}{8}\int_{\rho\geq 1} d^3\boldsymbol{\rho}\, [F_\pi^2 + \cos\chi \sin^2\nu\, (F_K^2 - F_\pi^2)] \times (1 + \cos^2\alpha) \sin^2\chi \tag{3.20}$$

$$b_s(\nu) = \frac{1}{8}\int_{\rho\geq 1} d^3\boldsymbol{\rho} \left[F_K^2 - (F_K^2 - F_\pi^2)\left(1 - \frac{1}{2}\cos\chi\right)\sin^2\nu\right] \times (1 - \cos\chi) \tag{3.21}$$

$$b_3(\nu) = \frac{1}{4}\int_{\rho\geq 1} d^3\boldsymbol{\rho}\, [F_\pi^2 + (F_K^2 - F_\pi^2)\cos\chi \sin^2\nu] \sin^2\chi \sin^2\alpha \tag{3.22}$$

$$b_J(\nu) = \frac{1}{8}\int_{\rho\geq 1} d^3\boldsymbol{\rho}\, [F_\pi^2 + (F_K^2 - F_\pi^2)\cos\chi \sin^2\nu] \times \left\{\zeta^2\left[\left(\frac{\partial\chi}{\partial\rho}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial\rho}\right)^2\right] + n^2\frac{\zeta^2}{\rho^2}\sin^2\chi\sin^2\alpha - 2\rho\zeta\left(\frac{\partial\chi}{\partial\rho}\frac{\partial\chi}{\partial\zeta} + \sin^2\chi\frac{\partial\chi}{\partial\rho}\frac{\partial\chi}{\partial\zeta}\right) + r^2\left[\left(\frac{\partial\chi}{\partial\zeta}\right)^2 + \sin^2\chi\left(\frac{\partial\alpha}{\partial\zeta}\right)^2\right]\right\} \tag{3.23}$$

The Hamiltonian (3.16) has a stable minimum with respect to the dimensional parameter ϵ when

$$\epsilon(\nu) = \left[\left\{\frac{\beta(\nu)}{2a_2(\nu)}\left[1 + \left(1 - \frac{4a_1(\nu)^3}{729\beta(\nu)a_2(\nu)^3}\right)^{1/2}\right] - \frac{a_1(\nu)^3}{729a_2(\nu)^3}\right\}^{1/3} + \left\{\frac{\beta(\nu)}{2a_2(\nu)}\left[1 - \left(1 - \frac{4a_1(\nu)^3}{729\beta(\nu)a_2(\nu)^3}\right)^{1/2}\right]\right.$$

$$- \frac{a_1(\nu)^3}{729a_2(\nu)^3}\Bigg\}^{1/3} - \frac{a_1(\nu)}{9a_2(\nu)}\Bigg]^{1/2} \tag{3.24}$$

where for the actual values of the quantities a_1, a_2, and β the constant cutoff is a positive quantity, i.e., $\epsilon > 0$.

The spectrum of the strange dibaryons is now given by (3.16) with ϵ given by (3.24). In the chirally symmetric case, i.e., when $m_K = m_\pi = 0$, we have $a_2(\nu) = 0$ and thus we obtain

$$\epsilon(\nu) = \left[\frac{3\beta(\nu)}{a_1(\nu)}\right]^{1/4}, \qquad H(\nu) = \frac{4}{3}\,[3a_1(\nu)^3\beta(\nu)]^{1/4} \tag{3.25}$$

and these are the familiar 1/4-power formulas obtained in (2.12) and (2.13). Furthermore, in the spherical single-baryon ($B = 1$) case, with both chiral and flavor symmetries unbroken, the results (3.25) are identical to the results (2.12) and (2.13) for the constant cutoff ϵ and energy of the system E, respectively. In other words in that case we have $a_1(\nu) \to a = 0.78$ GeV^2 and $\beta(\nu) \to (1/2b)J(J + 1)$ with $b = 0.91$ GeV^2 for $F_\pi = 186$ MeV.

3.4. Numerical Results

The numerical results for the spectrum of some strange dibaryon states ($B = 2$) for $F_\pi = 186$ MeV and $F_K = 226$ MeV and the empirical values for m_π and m_K are given in Table I. These results are classified in the same way as in Kopeliovich *et al.* (1992) and compared to the results obtained using the complete Skyrme model in that work. For a detailed discussion of the group-theoretic structure of the results presented in Table I, see Kopeliovich *et al.* (1992).

From Table I we see that, as in the spherically symmetric case, we obtain rather good agreement for the absolute masses of the states with relatively low (iso)spin and strangeness quantum numbers. For the states with higher (iso)spin and strangeness our results obtained using the approximately 1/4-power law are somewhat higher than those obtained using the complete Skyrme model. However, the results obtained here show reasonable qualitative agreement with the results obtained in Kopeliovich *et al.* (1992) even for the states with higher (iso)spin and strangeness.

4. CONCLUSIONS

The present paper has shown the possibility of using the Skyrme model to calculate the spectrum of strange dibaryon states as torus-like configurations without the use of the Skyrme stabilizing term, proportional to e^{-2}, which

Table I. Absolute Masses of some Strange Dibaryon States ($B = 2$) in GeV[a]

I	S	M	M^{KSS}
0	0	4.79	4.68
$\frac{1}{2}$	−1	5.01	4.92
1	−2	5.43	5.20
$\frac{3}{2}$	−3	6.03	5.47
1	0	4.82	4.72
$\frac{1}{2}$	−1	5.04	4.96
$\frac{3}{2}$	−1	5.19	5.05
0	−2	5.21	5.17
1	−2	5.36	5.24
2	−2	5.53	5.41
$\frac{1}{2}$	−3	5.77	5.41
$\frac{3}{2}$	−3	5.99	5.54
1	−4	6.06	5.63
2	0	4.85	4.88
$\frac{3}{2}$	−1	5.21	5.11
$\frac{5}{2}$	−1	5.45	5.29
1	−2	5.67	5.33
2	−2	5.89	5.49
$\frac{1}{2}$	−3	6.07	5.52
$\frac{3}{2}$	−3	6.17	5.65
0	−4	6.34	5.68
1	−4	6.39	5.78
$\frac{1}{2}$	−5	6.77	5.87
3	0	4.91	5.12
$\frac{5}{2}$	−1	5.29	5.36
2	−2	5.69	5.58
$\frac{3}{2}$	−3	6.06	5.78
1	−4	6.67	5.95
$\frac{1}{2}$	−5	6.86	6.09
0	−6	7.03	6.21

[a] M^{KSS} is the value according to Kopeliovich *et al.* (1992).

makes practical calculations more lengthy and painful and which is not sufficiently well understood in the framework of the underlying QCD.

The results for the absolute masses of the strange dibaryons, obtained using the empirical values of the parameters F_π, F_K, m_π, and m_K are in general qualitative agreement with those obtained using the complete Skyrme model. As in the $B = 1$ spherically symmetric case, we see that the difference between our results and those obtained using the complete Skyrme model increases with increasing (iso)spin and strangeness quantum numbers. This is a general feature of the 1/4-power law obtained as a consequence of the chiral quantum stabilization of the system.

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